Colloid Polym Sci 274:384–387 (1996)

ORIGINAL CONTRIBUTION

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# On the temperature distribution in the reverse Wilson chamber (RWC)

Received: 20 December 1994
Accepted: 19 September 1995

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**Abstract** An analytical expression for the so-called sensitivity time with account of the factors that govern it has been derived by means of the Kelvin method for propagation of heat waves in the Earth's core. Several factors (gas density, surface to volume ratio etc.) must be taken into account when designing a new reverse Wilson chamber.

**Key words** Reverse Wilson chamber – heterogeneous nucleation – thermal conductivity – sensitivity time

## Introduction

Both homogeneous and heterogeneous nucleation are taking place when a given change in volume in the corresponding chambers is reached. Thus, in the classical Wilson chamber (CWC) [1] the nucleation takes place in the bulk of chamber when the inert (noncondensing) gas–vapour mixture is suddenly expanded. Because of the fast expansion the gas phase is cooled while the walls of the chamber remain, however, for some time warmer, this time being termed sensitivity time, $t_s$. Evidently, the abovementioned circumstance is the reason the nucleation to take place in the bulk of chamber but not on the chamber walls.

The situation in the reverse Wilson chamber (RWC) is a "mirror" image of the physical picture described above [2–10]. The fast compression of the inert gas–vapour mixture causes warming of the gas phase, but the chamber walls remain colder for some time, the latter quantity being termed sensitivity time again.

It is evident that in this case the nucleation will take place on the chamber walls but not in the chamber bulk.

The sensitivity time is an important characteristics of both the classical and the reverse Wilson chamber method. In both cases the sensitivity time is strongly dependent on the chamber geometry as will be demonstrated in text.

The aim of this paper is to analyze the conditions, which determine the sensitivity time value, $t_s$, in the case of heterogeneous nucleation (RWC method).

## Temperature distribution in the reverse Wilson chamber

After the compression has been terminated the carrier (inert) gas–water vapour mixture becomes warmer, supersaturation is produced, and after reaching some value termed critical supersaturation rapid condensation occurs. The condensation takes place on the upper plate of the chamber, the inner surface being covered with a thin layer of an organic liquid. The use of organic liquid eliminates one of the usual objections against the solid substrates – i.e. solids do not have geometrically and energetically homogeneous surface – the condensation may take place on the so-called active sites.

A simple schematic of RWC is given in Fig. 1.

In the case of the classical Wilson cloud chamber the sensitivity time, $t_s$, is defined as a time lag during which the supersaturation preserves its value so that the nucleation on ions is still occuring (Williams [12]).

Hazen analyzed in details [11] this quantity ($t_s$) by using several methods and determined the factors that govern $t_s$ value. In the following we shall analyze the processes that take place in the RWC after the compression of the gas phase (inert gas + vapour) is terminated.

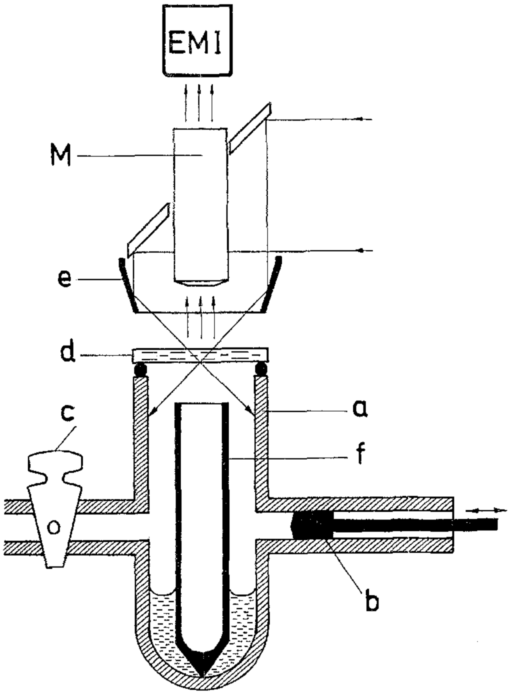

**Fig. 1** A simplified version of RWC given in [6]. M – light microscope; a – glass Chamber; b – piston by which supersaturation is produced, c – tap; d – removable chamber lid the inner surface of which is covered with a thin layer of the liquid substrate; e – objective; f – nontransperant ("black" Pt cylinder); More detailed description is given elsewhere [8]

Let the temperature of the carrier gas–vapour mixture be $T_f$ after compression and this temperature is greater than its initial value hereafter denoted as $T_i$ ($T_i < T_f$).

Because of the high thermal conductivity and molar heat capacity of the chamber walls (usually glass made) the walls of chamber preserve to some extent its initial temperature. Nevertheless, near the walls there exists a temperature gradient depending on several factors.

Our case is an analog of the so-called Kelvin's problem in solving the problem of propagation of heat waves in the Earth core [1, 13, 14].

The temperature $T_x$ at a distance $x$ from the chamber wall at time $t$ must be expressed in the following manner

$$T_x = T_f - (T_f - T_i)\frac{2}{(\pi)^{\frac{1}{2}}}\int_0^{\xi} \exp(-\xi^2)\, d\xi\,. \tag{1}$$

In the above formula

$$\xi = x/2ht^{1/2} \text{ and } h^2 = K/c\rho \tag{2}$$

where $K$ – heat conductivity, $c$ – molar heat capacity, $p$ – density, and $h^2$ is the diffusion coefficient.

We shall find the increment of the boundary layer due to temperature change.

For a layer with width $dx$ it equals

$S\,dx\,(\Delta T/T_i)$; $\Delta T = T_x - T_i$.

Integration in the limits specified below gives

$$d(\Delta V) = \frac{T_f - T_i}{T_i} S\left[1 - 2(\pi)^{-1/2}\int_0^{\xi}\exp(-\xi^2)\,d\xi\right]dx\,. \tag{3}$$

The total volume change is denoted by $\Delta V$ and can be obtained by means of expression (3) in the limits specified

$$\begin{aligned}\Delta V &= \frac{S(T_f - T_i)}{T_i}\\ &= 2ht^{1/2}\int_0^{\infty}\left[1 - 2(\pi)^{1/2}\int_0^{y}\exp(-\xi^2)d\xi\right]dy\\ &= 1.14\frac{S\,(T_f - T_i)}{T_i}\,ht^{1/2}.\end{aligned} \tag{4}$$

The boundary layer tends to compress this part of the fluid phase which is at the initial temperature. The change in temperature is thus

$$\begin{aligned}\Delta T &= (\gamma - 1)(\Delta V/V)T_i\\ &= 1.14\,(S/V)(T_f - T_i)\,ht^{1/2}(\gamma - 1)\,.\end{aligned} \tag{5}$$

Let $\Delta T_s$ be the limiting change in temperature after which the chamber becomes insensitive, then

$$t_s = \left[\frac{\Delta T_s}{T_f - T_i}\right]^2\left[\frac{V}{S}\right]^2\left[\frac{\rho c}{K}\right]\left[\frac{1}{\gamma - 1}\right]^2\left[\frac{1}{1.14}\right]^2. \tag{6}$$

In Eqs. (3–6) $S$ is the chamber's surface and $\gamma = C_p/C_v$, where $C_p$ and $C_v$ are the molar heat capacities at constant pressure and constant volume, respectively.

Only the value of $\Delta T_s$ is to be determined. Let $1 + \varepsilon$ is the compression at which vapour nucleates on the substrate surface, and $1 + \varepsilon + \Delta\varepsilon$ is the maximum compression at which no fluid droplets are formed.

Thus, one may write (it follows directly from the adiabatic Poisson's law), taking into account that the volume in the initial state is $V + \Delta V$, the volume in the final state (after compression) being $V$

$$T_i/T_f = (1 + \varepsilon)^{(\gamma - 1)} \tag{7}$$

and, therefore, taking logarithm, the following approximate expression holds

$$(T_f - T_i)/T_i = (\gamma - 1)\varepsilon = T/T_i\,, \tag{8}$$

where $T$ is the difference in temperature. In Eqs. (7) and (8) $\varepsilon$ is equal to $\Delta V/V$.

Therefore,

$$\Delta T/T = \Delta\varepsilon/\varepsilon \tag{9}$$

and one may obtain the following formula for $t_s$

$$t_s = \left[\frac{\Delta\varepsilon}{\varepsilon}\right]^2 \left[\frac{V}{S}\right]^2 \left[\frac{\rho C}{K}\right] \left[\frac{1}{\gamma - 1}\right]^2 0.77. \tag{10}$$

## Factors that govern the $t_s$ value

The factors that govern the $t_s$ value are five:

1) gas density $\rho$
2) $(\Delta\varepsilon/\varepsilon)^2$
3) Supersaturation $\Sigma$.

This factor does not enter directly in Eq. (10) but it actually affects the value of $t_s$ since for perfect gases the supersaturation is determined through the Poisson equation and therefore the supersaturation, $S$, may be expressed in the following form

$$\Sigma = (1 + \varepsilon)^\gamma \tag{11}$$

4) Square of the volume-to-surface ratio, $(V/S)^2$
5) $\gamma = C_p/C_v$– ratio

This dependence has two aspects:

(a) $\gamma$ is different for different carrier gases and condensing liquids

(b) $\gamma$ is dependent upon the total pressure in the chamber, $P_t$. through the Richarz–Powell equation [14, 15], which relates the pressures of the carrier gas $P_s$ and vapour $P_v$, and the corresponding $\gamma$-ratios, $\gamma_g$ and $\gamma_v$, respectively,

$$\frac{1}{\gamma - 1} = \frac{1}{\gamma_g - 1}\frac{P_g}{P_t} + \frac{1}{\gamma_v - 1}\frac{P_v}{P_t} \tag{12}$$

where $P_t = P_g + P_v$ (for perfect gases) is the total pressure of the carrier gas–vapour mixture and $\gamma$ is the mixture's $C_p/C_v$-ratio.

Therefore, the change in any of the quantities listed above will result in corresponding change in $\gamma$.

## Examples for the applicability of equation (10)

Dependence of $t_s$ on $\gamma$

Two chambers with $V/S$ ratio 3.21 and 4.1 respectively have been used in our studies [12–10]. In these experiments pure argon and air as carrier gases are employed. We shall analyze the influence of $y$ on the basis of the following example. $\gamma_{ar}$ and $\gamma_{air}$ are 1.667 and 1.401, respectively [16]. $\gamma_v = 1.35$ [16]. In both experiments in question $V/S = 3.21$ and $P_t = 742.5$ mm Hg, $P_v = 23.76$ mm Hg, $P_{ar} = 718.74$ mm Hg, and $P_{air} = 718.74$ mm Hg. Richarz–Powell equation gives $\gamma_{ar} = 1.643$ and $\gamma_{air} = 1.390$. In both cases $\Delta\varepsilon/\varepsilon = 0.219$, $\rho_{ar} = 1.76 \times 10^{-3}$ g/cm$^3$, $C_{ar} = 0.125$ cal/g deg, and $K = 4.4 \times 10^{-3}$ cal/g deg [10]. The corresponding values for air are (in the same units); $1.1608 \times 10^{-3}$, 1.006, and $6.194 \times 10^{-5}$, respectively [16]. Thus, $t_s = 0.401$ (for argon) and 0.109 (for air). Hereafter all values of $t_s$ are in seconds. It is evident that after 0.4 s the chamber becomes insensitive. This is in accordance with the calibration curve for Ar given in Appendix.

Dependence on supersaturation ln $\Sigma$

Such a dependence appears because the supersaturation is related to $\varepsilon$ by $\varepsilon = 1 - \exp(\ln \Sigma/\gamma)$. We shall not give any examples since the calculation by means of formula (10) is straightforward.

Dependence of $t_s$ on $V/S$ ratio

For the chambers with $V/S$ equal to 3.21 and 4.1 we obtain for the sensitivity time 0.4 s and 0.74 s, respectively.

We shall note that Hazen [11] and Williams [12] performed such measurements in the classical Wilson chamber varying the geometry of the system and their results are similar to those reported here.

It should be noted that Jolliot [13] has performed experiments with a classical Wilson chamber on the dependence of $t_s$ as a function of the gas density $\rho$. His results are in good agreement with Eq. (10)

## Conclusion

A simple formula for the so-called sensitivity time for the case of the reverse Wilson chamber has been derived by using the Kelvin method. According to this equation several factors may affect the sensitivity time-supersaturation, $\gamma$, total pressure, $V/S$ ratio, etc. The sensitivity time calculated for argon is in good agreement with the experimental data. The results obtained demonstrate that the boundary layer in the chamber is quite narrow so that there is sufficient time for nucleation. The geometry of the chamber must be taken into account when designing new chamber in order to improve its sensitivity characteristics and the reliability of the results obtained.

## Appendix

Deviations from the adiabaticity in RWC

It is accepted in earlier papers [2, 3] that the regime in the chamber is adiabatic one, following an analogy with the

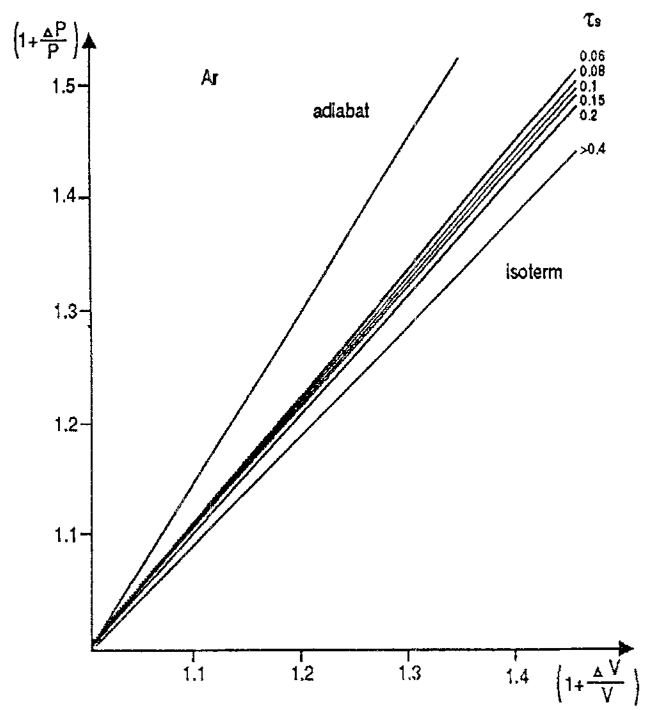

**Fig. 2** Calibration curves for very pure Ar at different compression times $\tau$

classical Wilson chamber [1]. However, it was established in [8] that the regime of compression of the carrier gas-vapour mixture in the case of RWC is intermediate between isothermal and adiabatic one. The deviation from the adiabaticity are dependent upon the compression time $\tau$ [8]. Let $V + \Delta V$ be the initial volume of the chamber and $V$ be the final one. Then, the supersaturation $\ln \Sigma$ is to be determined by making use of the Poisson law $pV^{\gamma} = \text{const}$

$$\ln \Sigma = \ln(1 + V/V)^{\gamma} \qquad \text{(A.1)}$$

if the compression in the chamber proceeds as an adiabatic process.

In the case of nonadiabatic process the equation of polytrope holds $pV^{\gamma}$ eff = const where $\gamma_{\text{eff}}$ lie in the interval between 1 (isothermal) and $\gamma$ (adiabatic) [3, 4].

Therefore, the Poisson equation cannot be used for determination of the supersaturation $\ln \Sigma$.

Chakarov et al. [8] investigated the changes in pressure as a function of the volume change at different compression times. They presented the experimental results in coordinates $(1 + \Delta p_{\text{m}}/p_{\text{at}})$ vs. $(1 + \Delta V/V)$ where $\Delta p_{\text{m}}$ is the maximum change in pressure and $p_{\text{at}}$ is the atmospheric pressure.

The calibration curves of carrier gas very pure *Ar* in the above mentioned coordinates is given in Fig. 2.

As can be seen from this figure at $\tau \geq 0.4$ s, the pressure increases from its initial value (atmospheric pressure) to its final value $(p_{\text{at}} + \Delta p_{\text{is}})$ where $p_{\text{is}}$ is the relative change in $p$ due to isothermal compression [8].

Therefore, one may write,

$$p_{\text{at}}(V + \Delta V) = (p_{\text{at}} + p_{\text{is}}) \quad \text{or} \quad p_{\text{is}} = p_{\text{at}}(V + \Delta V)\,. \qquad (A.2)$$

Consequently, the faster is the compression, more rapid is the pressure change [4, 8]. Because of this one may obtain for the supersaturation

$$\ln \Sigma = \ln(1 + \Delta p_{\text{m}}/p_{\text{at}}) \qquad \text{(A.3)}$$

as is done in [4, 8].

If $\tau \geq 0.4$ s (slow compression) no condensation takes place [3, 4]. This experimental finding is in accordance with the above presented results because the sensitivity time for Ar is 0.4 sec again.

Finally, one may conclude that in Eq. (10) and all other equations involving $\gamma$ the latter quantity must be replaced by an effective value taken from the experiments [3, 4].

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